CSC 742
Database Management Systems
Topic #8:
Relational Calculus
Relational Calculus: 1

- Can define the information to be retrieved
  - not any specific series of operations
  - In contrast, relation algebra defines the sequence of operations.
- Closer than relational algebra to how users would formulate queries
  - in terms of their information needs, rather than in terms of operations.
Relational Calculus: 2

- Involves variables that range over tuples or domain attributes
  - **Tuple Relational Calculus**: Variables range over tuples
  - **Domain Relational Calculus**: Variables range over domain attributes
- Gives a way of referring to attributes of relations (and the specific tuples in them)
- Is based on predicate logic—gives the usual quantifiers to construct complex queries.
Tuple Relational Calculus (TRC)

- TRC is based on tuple variables.
- Each tuple variable ranges over a relation.
  - The relation is called the range relation of the variable.
- Examples:
  - \{t \mid \text{Employee}(t) \text{ and } t.\text{salary} > 50000\}
    - Retrieve all attribute values.
  - \{t.\text{Fname}, t.\text{Lname} \mid \text{Employee}(t) \text{ and } t.\text{salary} > 50000\}
    - Retrieve selected attributes.
### Employee

<table>
<thead>
<tr>
<th>SSN</th>
<th>Lname</th>
<th>Fname</th>
<th>Salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>111-22-3333</td>
<td>Smith</td>
<td>John</td>
<td>30000</td>
</tr>
<tr>
<td>121-23-3333</td>
<td>Wong</td>
<td>Frank</td>
<td>45000</td>
</tr>
<tr>
<td>153-32-1342</td>
<td>Wallace</td>
<td>Jennifer</td>
<td>43000</td>
</tr>
<tr>
<td>154-33-3333</td>
<td>Borg</td>
<td>James</td>
<td>56000</td>
</tr>
<tr>
<td>555-44-5555</td>
<td>English</td>
<td>Joyce</td>
<td>53000</td>
</tr>
</tbody>
</table>

\{ t | Employee(t) and t.salary > 50000 \}

\{ t.Fname, t.Lname | Employee(t) and t.salary > 50000 \}
TRC (Cont’d)

■ To specify a TRC expression:
  ◆ Specify the range relation R of each tuple variable t.
    ♦ In the form of R(t).
  ◆ Specify a condition to select particular combinations of tuples.
  ◆ Specify a set of attributes to be retrieved.

\{t.Fname, t.Lname | Employee(t) and t.salary > 50000\}
Employee

<table>
<thead>
<tr>
<th>SSN</th>
<th>Lname</th>
<th>Fname</th>
<th>Salary</th>
</tr>
</thead>
<tbody>
<tr>
<td>111-22-3333</td>
<td>Smith</td>
<td>John</td>
<td>30000</td>
</tr>
<tr>
<td>121-23-3333</td>
<td>Wong</td>
<td>Frank</td>
<td>45000</td>
</tr>
<tr>
<td>153-32-1342</td>
<td>Wallace</td>
<td>Jennifer</td>
<td>43000</td>
</tr>
<tr>
<td>154-33-3333</td>
<td>Borg</td>
<td>James</td>
<td>56000</td>
</tr>
<tr>
<td>555-44-5555</td>
<td>English</td>
<td>Joyce</td>
<td>53000</td>
</tr>
</tbody>
</table>

1. Retrieve the salary of ‘John Smith’.
2. Retrieve the names of the employees whose salary is less than 40000.
General TRC expression

- \{ t_1.A, ..., t_n.A_n \mid COND(t_1, ..., t_{n+m}) \}
  - COND \rightarrow COND \& COND
  - COND \lor COND
  - \sim COND
  - ATOM
  - (\forall t: COND)
  - (\exists t: COND)

- Forms of ATOMs
  - R(t)
  - Ti.A \ op Tj.B, where \ op \in \{=, >, \geq, <, \leq, \neq\}.
  - Ti.A \ op c, where \ op \in \{=, >, \geq, <, \leq, \neq\} and c is a constant.
Tuple Variables

- Tuple variables in a COND may be
  - bound: within the scope of a quantifier
  - free: not within the scope of a quantifier
- Intuitively, the COND describes a property of its free variables
- All and only the free variables of COND can show up in the answer tuple as specified in TRC
\{t.\text{Fname}, t.\text{Lname} \mid \text{Employee}(t) \text{ and } t.\text{salary} > 50000\} \\

\{t.\text{Fname}, t.\text{Lname} \mid \text{Employee}(t) \text{ and } t.\text{salary} > 50000 \text{ and} \text{Department}(d) \text{ and } d.\text{MgrSSN} = t.\text{SSN}\} \\

\{t.\text{Fname}, t.\text{Lname} \mid \text{Employee}(t) \text{ and } t.\text{salary} > 50000 \text{ and} \exists d (\text{Department}(d) \text{ and } d.\text{MgrSSN} = t.\text{SSN})\}
Conventions

- Nested forall expressions can be simplified:
  \((\forall t1: (\forall t2: C)) = (\forall t1, t2: C)\)

- Nested exists expressions can be simplified:
  \((\exists t1: (\exists t2: C)) = (\exists t1, t2: C)\)

- For safety (described below), all tuple variables are declared to belong to some relation, e.g., project(p1) or employee(e2)
TRC Examples: 1

- Find names of employees in the research dept

Employee(Fname, Lname, SSN, Bdate, Address, Sex, Salary, SuperSSN, Dno)

Department(Dname, Dnumber, MgrSSN, MgrStartDate)
TRC Examples: 2

- For every project in 'Stafford' list the controlling dept number and the dept manager's last name

Employee(Fname, Lname, SSN, Bdate, Address, Sex, Salary, SuperSSN, Dno)

Department(Dname, Dnumber, MgrSSN, MgrStartDate)

Project(Pname, Pnumber, Plocation, Dnum)
TRC Examples: 3

- Find SSNs of employees who work on all the projects controlled by dept 5.

Employee(Fname, Lname, SSN, Bdate, Address, Sex, Salary, SuperSSN, Dno)

Works_on(ESSN, Pno, Hours)

Project(Pname, Pnumber, Plocation, Dnum)
Using ER Diagrams

ER diagrams can be used to understand and formulate queries

- Paths
  - correspond to constraints on what is being retrieved
  - can be mapped to calculus expressions
  - encapsulate some common English grammatical constructions
Find SSNs of employees who work on all the projects controlled by dept 5.

Employee(Fname, Lname, SSN, Bdate, Address, Sex, Salary, SuperSSN, Dno)

Works_on(ESSN, Pno, Hours)

Project(Pname, Pnumber, Plocation, Dnum)
Safe TRC Expressions

- A Safe TRC expression:
  - One that is guaranteed to yield a finite number of tuples as its result.

- An unsafe TRC expression
  - \{ t \mid \neg (Employee (t)) \}
Safe TRC Expressions (Cont’d)

- Domain of a TRC expression:
  - set of all values that appear in the expression or in any tuple of any relation mentioned in the expression
  - Example: \{ t | not (Employee (t)) \}

- Safe expressions:
  - those whose result includes values only from their own domain
  - \{ t | Employee(t) & ... \} is safe because the answer is a subset of the Employee relation
  - \{ t | not (Employee (t)) \} is not safe because …
Safe TRC Expressions (Cont’d)

- Outer-level negations are unsafe
- Interestingly, even disjunctions can lead to trouble:
  - \( \{ \langle x, y \rangle \mid R(x) \lor S(y) \} \) returns tuples whose first member is in relation \( R \) or whose second member is in relation \( S \). Suppose \( x_1 \) is in \( R \). Then the result contains \( \langle x_1, y \rangle \) for every \( y \), irrespective of whether \( y \) is in \( S \). Hence this query is not safe.
Safe TRC Expressions (Cont’d)

Unsafe expressions are undesirable because they can involve

- infinite answer sets
- answers using values that the user (who produced the query) may have no way of predicting.
Limitations of TRC

- It cannot express queries involving
  - aggregations
  - closure
- It cannot express non-query operations
  - insert
  - delete
  - update
because those involve modeling the changes in the state of a database.